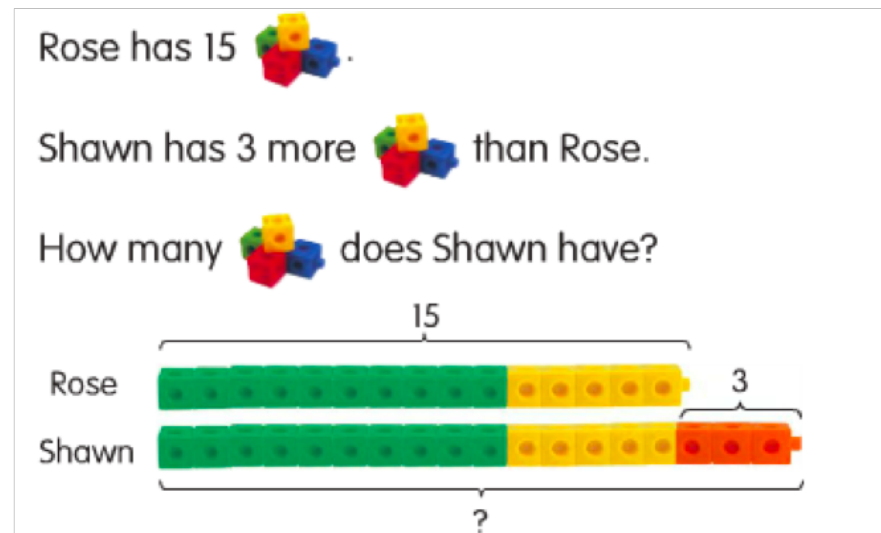


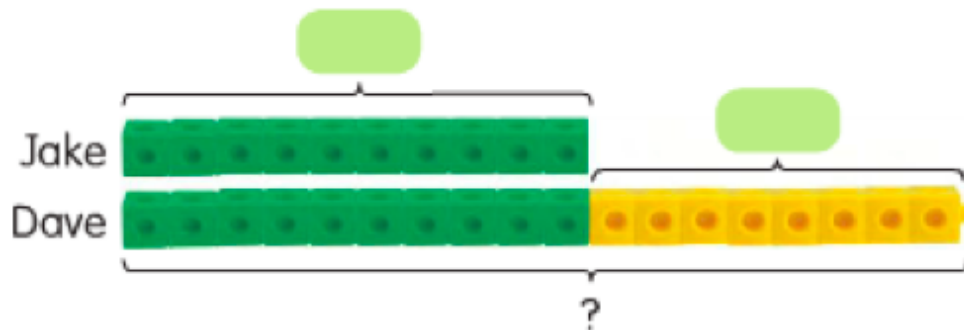
# Bar Modeling

## A Visual Approach to Word Problems

In Kindergarten and 1st grade, bar modeling is not explicitly taught, but we begin the concept by using comparative language and connecting cubes as a concrete model to represent quantities.



Jake makes 10 glasses of orange juice.  
Dave makes 8 more glasses of orange juice than Jake.  
How many glasses of orange juice does Dave make?

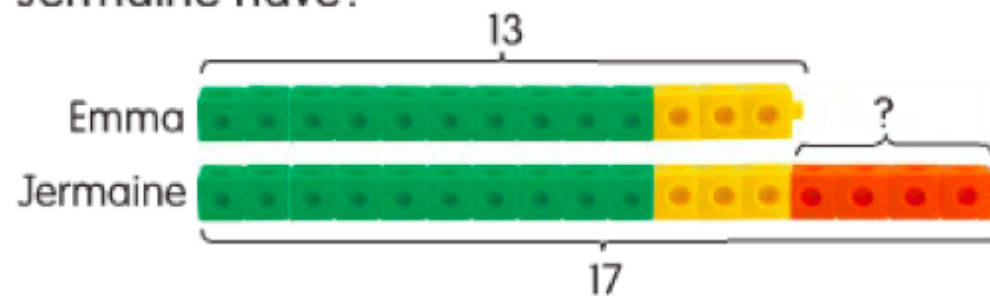


$$\boxed{\phantom{00}} \bigcirc \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

Dave makes  glasses of orange juice.

Emma has 13 stickers.  
Jermaine has 17 stickers.  
How many more stickers does  
Jermaine have?

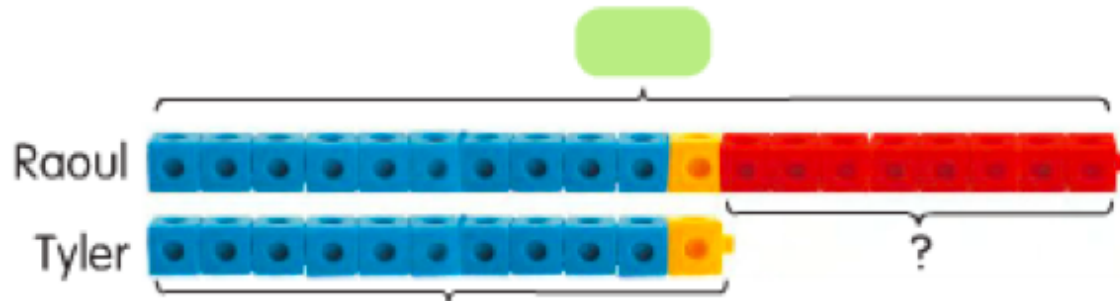
Use  to show the  
number of stickers.



Raoul has 19 baseball cards.

Tyler has 11 baseball cards.

How many more baseball cards does Raoul have?



$$\text{ } \text{ } \text{ } = \text{ }$$

Raoul has  more baseball cards.

## 1st Grade Enrichment

Wendy has 5 more dolls than Jill.

Jill has 6 fewer dolls than Nellie.

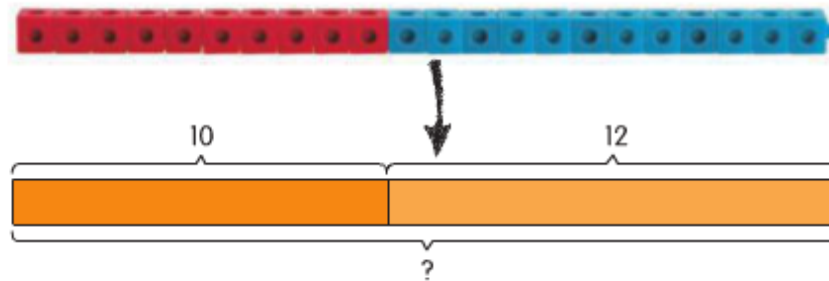
Kiri has 3 more dolls than Jill.

Nellie has 9 dolls.

How many dolls do the four girls have in all?

## 2nd Grade students are introduced to model drawing to represent part/part/whole relationships.

Mandy makes 10 granola bars.  
Aida makes 12 granola bars.  
How many granola bars do they make in all?



$$10 + 12 = 22$$

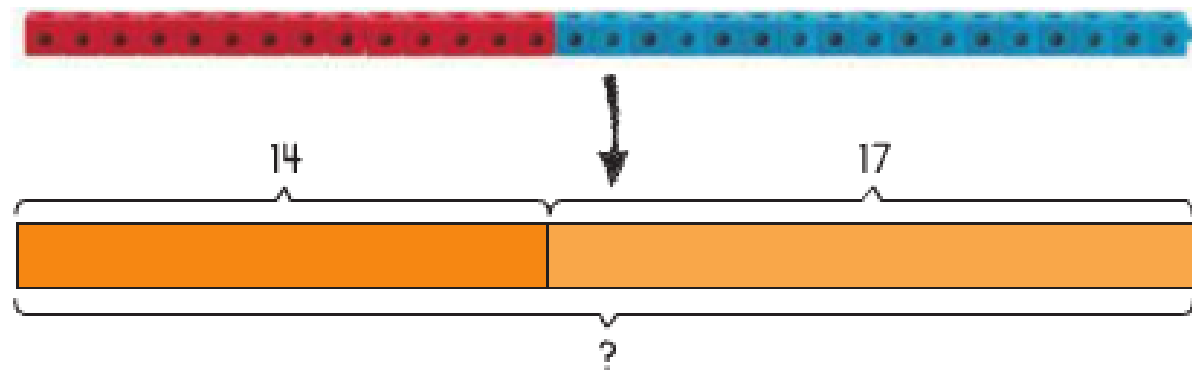
They make 22 granola bars in all.

**Check!**

$22 - 10 = 12$   
 $22 - 12 = 10$   
The answer is correct.



Helen puts 14 breadsticks in a basket.  
Her friend puts 17 breadsticks in the basket.  
How many breadsticks are in the basket?

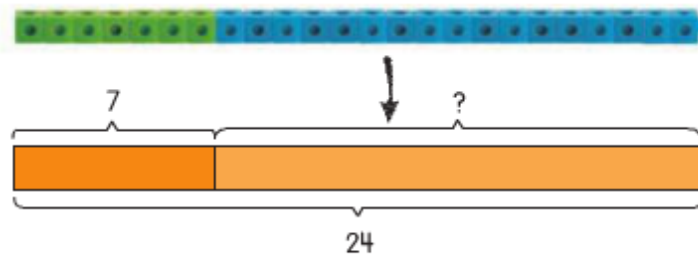


$$\text{ } + \text{ } = \text{ }$$

There are  breadsticks in the basket.

**Students learn that when you are missing a part, you have to subtract a part from the whole to get the other part.**

Will buys 24 eggs.  
He breaks 7 eggs.  
How many eggs do not break?



$$24 - 7 = 17$$

17 eggs do not break.



**Check!**

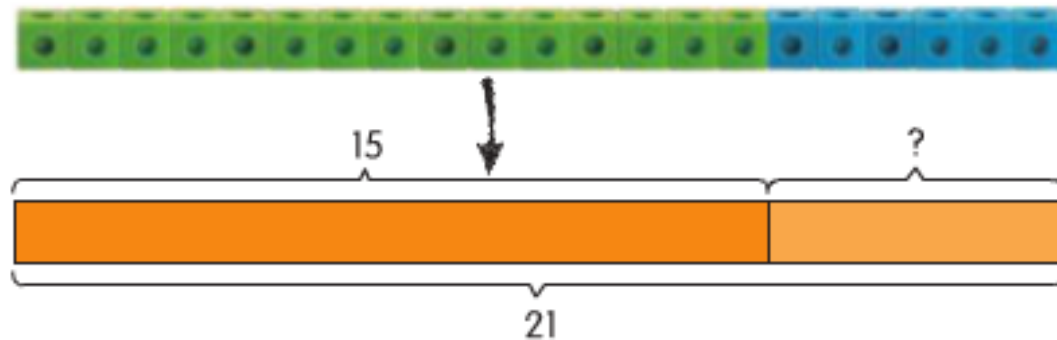
$$17 + 7 = 24$$

The answer is correct.





The second grade class has a new aquarium.  
There are 21 fish in it.  
15 fish were given by families.  
The rest were bought by the school.  
How many fish did the school buy?



$$\text{ } - \text{ } = \text{ }$$

The school bought  fish.

Check!

$$\text{ } + \text{ } = \text{ }$$

Is the answer correct?

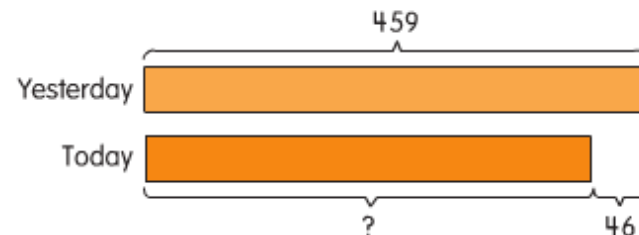


**Beginning in 2nd grade and continued in 3rd grade students learn to use the comparison bar model. Students learn to draw one bar above another and represent bars as longer or shorter.**

**Learn**

**You can use bar models to show comparing sets to subtract.**

459 children were at the library yesterday.  
46 fewer children are at the library today.  
How many children are at the library today?



$$459 - 46 = 413$$

413 children are at the library today.

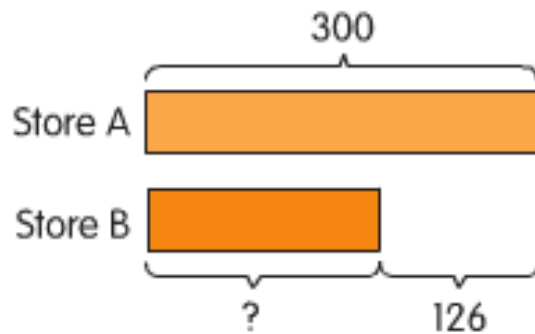
**Check!**

$$413 + 46 = 459$$

The answer is correct.

In Store A, 300 video games are sold.  
This is 126 more games sold than in Store B.  
How many video games are sold in Store B?

It is important to not  
teach key words.



Check!

$$\text{ } + \text{ } = \text{ }$$

Is your answer correct?

$$\text{ } - \text{ } = \text{ }$$

video games are sold in Store B.



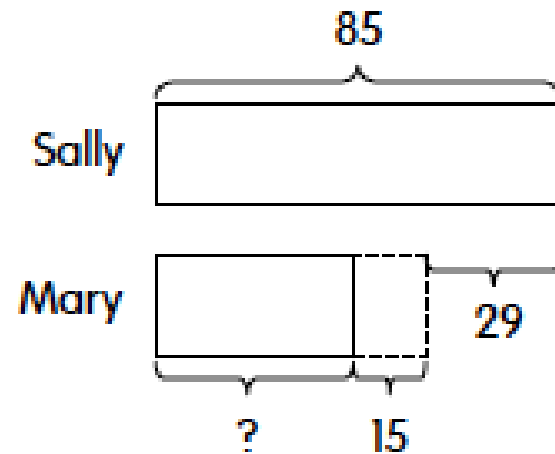
## 2nd Grade Enrichment

Sally has 85 stamps.

Mary has 29 stamps less than Sally.

Mary gives away 15 stamps.

How many stamps does Mary have in the end?



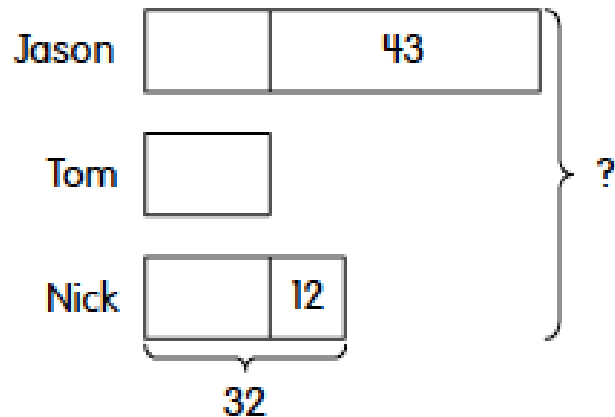
## 2nd Grade Enrichment

For a charity drive, Jason collects 43 more toys than Tom.

Tom collects 12 fewer toys than Nick.

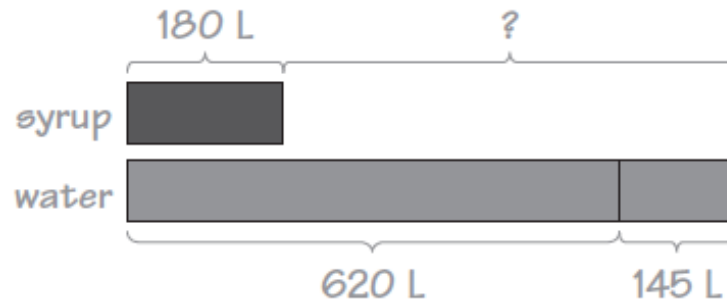
Nick collects 32 toys.

How many toys do the three children collect in all?



## 3rd Grade Real World Problem

Jake mixes 620 liters of water and 180 liters of syrup to make lemonade. He adds another 145 liters of water to the mixture. How much more water than syrup does he use for the lemonade?



$$620 + 145 = 765$$

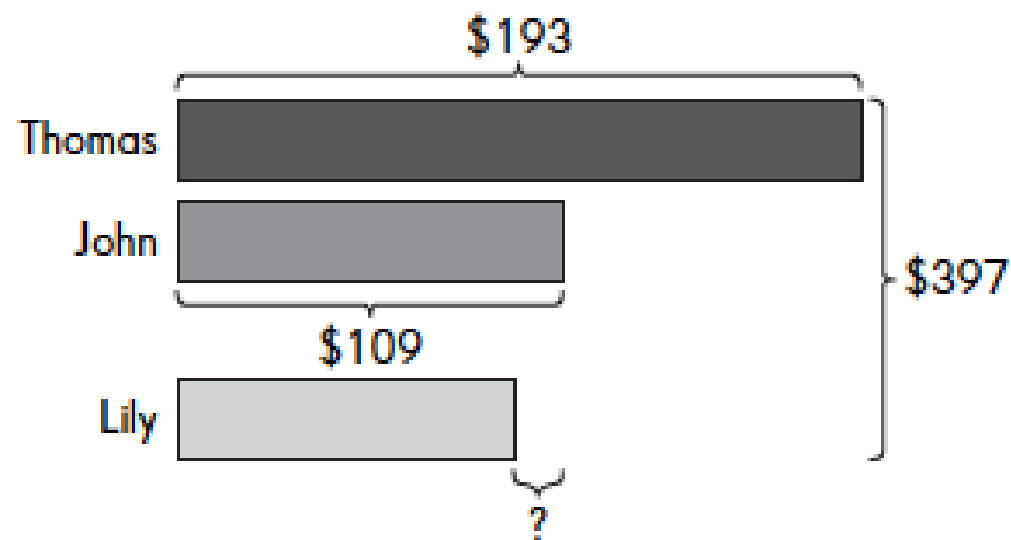
He uses 765 liters of water.

$$765 - 180 = 585$$

He uses 585 liters more water than syrup.

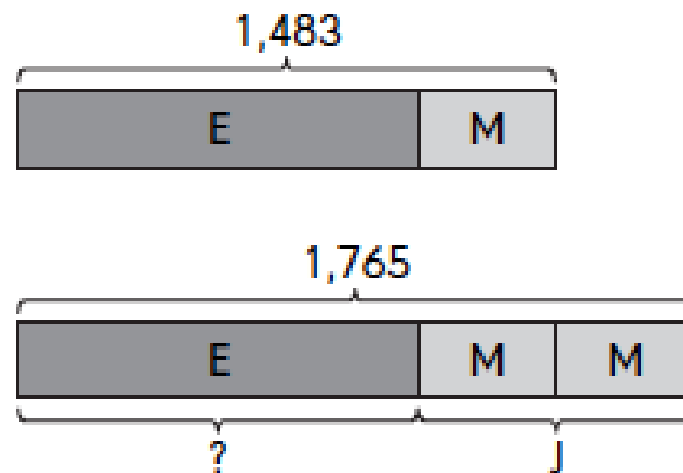
## 3rd Grade Enrichment

John, Thomas, and Lily save \$397 in all.  
John saves \$109 and Thomas saves \$193.  
Lily saves the least.  
How much more does John save than Lily?



## 3rd Grade Enrichment

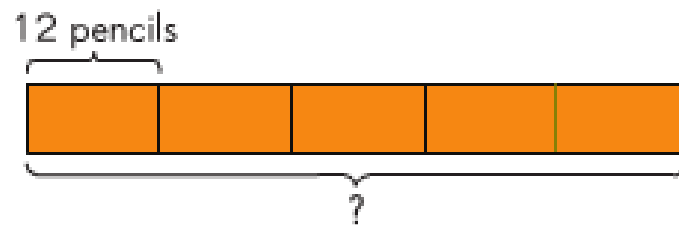
Emma and Mary have 1,483 baseball cards in all.  
Emma and Jennifer have 1,765 baseball cards in all.  
Jennifer has twice as many baseball cards as Mary.  
How many baseball cards does Emma have?





**Beginning in 2nd grade and continued in 3rd grade students learn to use the bar model for multiplication and division. These concepts are based on equal groups or the part-whole concept, where each equal group is one part of a whole.**

There are 5 boxes of pencils.  
Each box contains 12 pencils.  
How many pencils are there in all?



1 unit  $\longrightarrow$  12

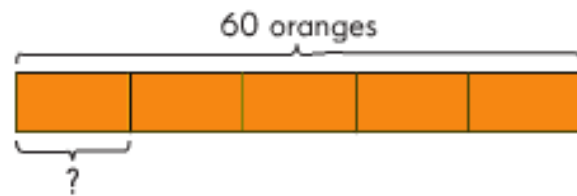
5 units  $\longrightarrow$   $12 \times 5 = 60$

There are 60 pencils in all.

A grower picks 60 oranges.

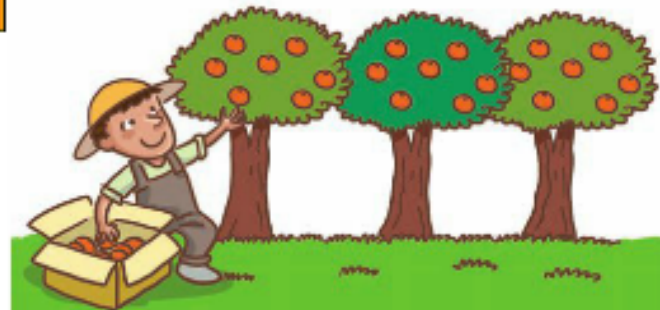
The grower packs them equally into 5 boxes.

How many oranges does the grower pack in each box?

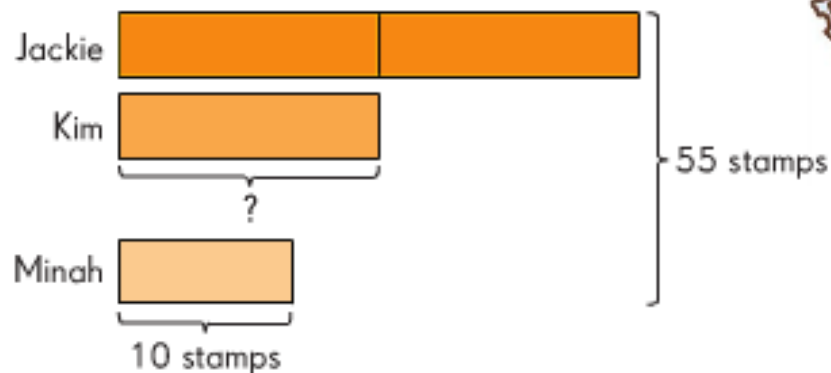


$$60 \div 5 = 12$$

He packs 12 oranges in each box.



Jackie, Kim, and Minah have 55 stamps in all.  
 Jackie has twice as many stamps as Kim.  
 Minah has 10 stamps.  
 How many stamps does Kim have?



$$\boxed{\phantom{00}} - \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

Jackie and Kim have  $\boxed{\phantom{00}}$  stamps.

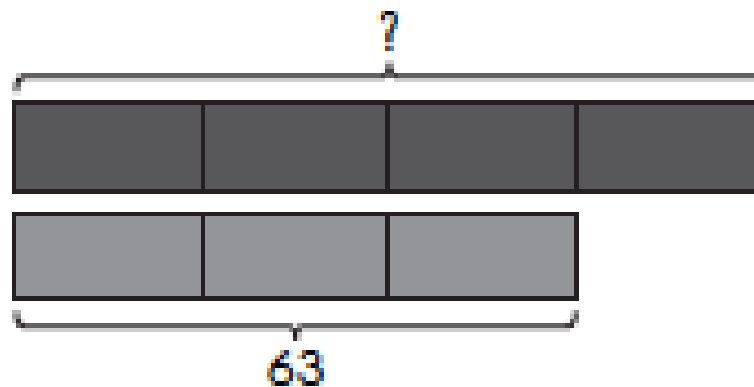
3 units  $\longrightarrow$   $\boxed{\phantom{00}}$

1 unit  $\longrightarrow$   $\boxed{\phantom{00}} \div 3 = \boxed{\phantom{00}}$

Kim has  $\boxed{\phantom{00}}$  stamps.

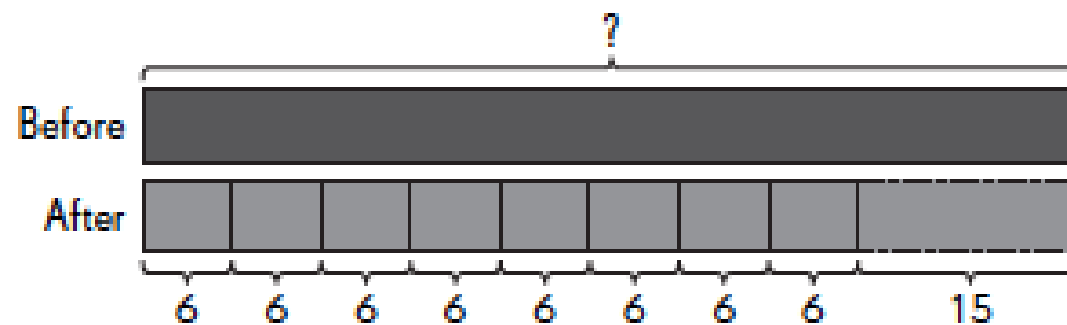
## 3rd Grade Enrichment

A number is divided by 4.  
The result is the same as 63 divided by 3.  
What is the number?



## 3rd Grade Enrichment

A fruit seller has some peaches.  
He throws away 15 rotten peaches.  
He packs the rest equally into 8 bags.  
There are 6 peaches in each bag.  
How many peaches did the seller have at first?



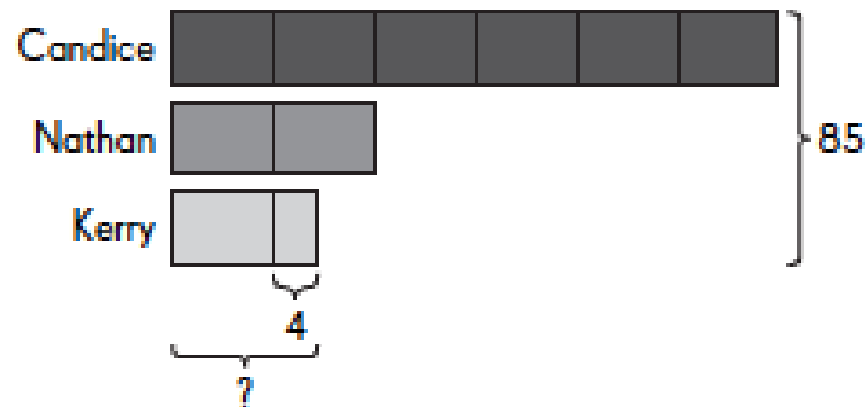
## 3rd Grade Enrichment

Candice, Kerry, and Nathan sold a total of 85 tickets to the school's production.

Candice sold three times as many tickets as Nathan.

Kerry sold 4 more tickets than half of what Nathan sold.

How many tickets did Kerry sell?



**In 4th grade and 5th grade students will use the bar model as a helpful tool in solving more complex problems.**

Michelle has some stamps.

Her sister has half as many stamps as she does.

They have 1,080 stamps in all. How many stamps does Michelle have?

## 4th Grade Real World Problem

Charlie has 5 times as many stamps as Ryan. They have 1,608 stamps in all.  
How many more stamps does Charlie have than Ryan?



**In 4th and 5th grade, students use models to understand and solve problems that involve fractions and proportional thinking. These problems involve part-whole relationships.**

Three friends shared a grapefruit.

Elena ate  $\frac{1}{3}$  of the grapefruit.

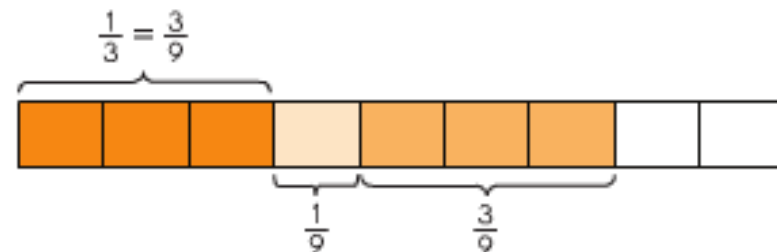
Lee ate  $\frac{1}{9}$  of the grapefruit.

Sara ate  $\frac{3}{9}$  of the grapefruit.

What fraction of the grapefruit did they eat altogether?

$$\begin{aligned}\frac{1}{3} + \frac{1}{9} + \frac{3}{9} &= \frac{3}{9} + \frac{1}{9} + \frac{3}{9} \\ &= \frac{7}{9}\end{aligned}$$

Elena, Lee, and Sara ate  $\frac{7}{9}$  of the grapefruit.



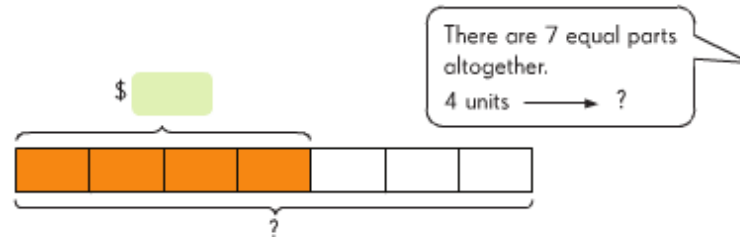
$$\frac{1}{3} = \frac{3}{9} \bigcirc \bigcirc$$



# 4th Grade Real World Problem

Vincent spent  $\frac{4}{7}$  of his money on a pair of shoes.

The shoes cost \$48. How much money did he have at first?



4 units  $\longrightarrow$  \$   

1 unit  $\longrightarrow$  \$     $\div$  4 = \$   

7 units  $\longrightarrow$  \$     $\div$  4 = \$   

He had \$    at first.



# 4th Grade Put On Your Thinking Cap

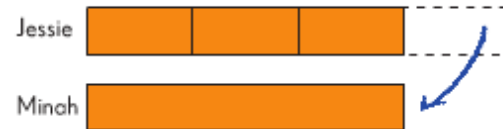
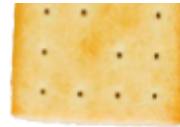
Jessie had a whole graham cracker.

Minah had only part of another graham cracker.

Jessie gave  $\frac{1}{4}$  of her graham cracker to Minah.

In the end, both girls had the same fractional part of a graham cracker.

What fraction of a graham cracker did Minah have at first?



Here are 2 equal bars to show that both of them had an equal portion of a graham cracker in the end.

Work backward to find the fraction of the graham cracker Minah had at first.

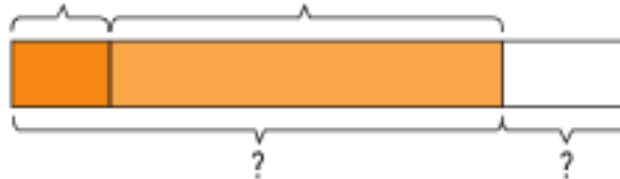


## 5th Grade Real World Problem

This is an application of the part-part-whole relationship which was developed since kindergarten.

Megan spent  $\frac{1}{6}$  of her money on food and  $\frac{5}{8}$  of her money on a new outfit. What fraction of Megan's money is left?

$$\frac{1}{6} = \frac{4}{24} \quad \frac{5}{8} = \frac{15}{24}$$



First, find the amount of money Megan spent on food and the new outfit.

$$\frac{1}{6} + \frac{5}{8} = \frac{19}{24}$$

Megan spent  $\frac{19}{24}$  of her money on food and the new outfit.

$$1 - \frac{19}{24} = \frac{5}{24}$$

$\frac{5}{24}$  of Megan's money is left.



## 5th Grade Real World Problem

**Learn** Find fractional parts of a whole and the remainder.

Sofia has \$480. She uses  $\frac{1}{3}$  of the money to buy a winter coat. She then spends  $\frac{1}{4}$  of the remainder on a pair of winter boots. How much money does she have left?

### Method 1

$$1 - \frac{1}{3} = \frac{2}{3}$$

Remainder  $\rightarrow$  2 parts

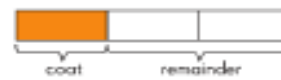
To show  $\frac{1}{4}$  of the remainder is spent on the boots, I have to further divide the remainder into 4 parts.

Least common multiple of 2 and 4 = 4

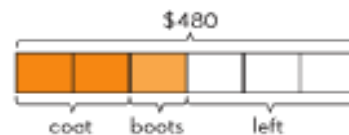
By equivalent fractions:

$$\frac{2}{3} = \frac{4}{6}$$

$\begin{array}{c} \nearrow \times 2 \\ \searrow \times 2 \end{array}$



I need to draw a model with 6 equal units to show the problem.



$$\frac{1}{3} \text{ of } 6 \text{ units} = 2 \text{ units}$$

$$\frac{1}{4} \text{ of } 4 \text{ units} = 1 \text{ unit}$$

The model shows that:

$$6 \text{ units} \rightarrow \$480$$

$$1 \text{ unit} \rightarrow \$480 \div 6 = \$80$$

$$3 \text{ units} \rightarrow 3 \times \$80 = \$240$$

She has \$240 left.



**Method 2**

$$\begin{aligned}\frac{1}{3} \text{ of } 480 &= \frac{1}{3} \times 480 \\ &= 160\end{aligned}$$

Sofia spends \$160 on the coat.

$$480 - 160 = 320$$

After buying the coat, she has \$320 left.

$$1 - \frac{1}{4} = \frac{3}{4}$$

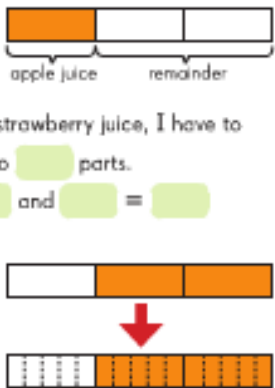
$$\begin{aligned}\frac{3}{4} \text{ of } 320 &= \frac{3}{4} \times 320 \\ &= 240\end{aligned}$$

She has \$240 left.

## 5th Grade Real World Problem

Jermaine prepares a mixture of apple, carrot, and strawberry juices. Of the total amount,  $\frac{1}{3}$  of the mixture is apple juice.  $\frac{2}{5}$  of the remainder is strawberry juice. Jermaine uses 315 milliliters of strawberry juice in the mixture. What amount of the mixture is carrot juice?

$1 - \frac{1}{3} = \frac{\quad}{\quad}$   
 Remainder  $\rightarrow$   $\frac{\quad}{\quad}$  parts  
 To show  $\frac{2}{5}$  of the remainder is strawberry juice, I have to further divide the remainder into  $\frac{\quad}{\quad}$  parts.  
 Least common multiple of  $\frac{\quad}{\quad}$  and  $\frac{\quad}{\quad} = \frac{\quad}{\quad}$   
 By equivalent fractions:  
 $\frac{\quad}{\quad} \xrightarrow{\times 5} \frac{\quad}{\quad} = \frac{\quad}{\quad} \xrightarrow{\times 5} \frac{\quad}{\quad}$   
 I need to draw a model with  $\frac{\quad}{\quad}$  equal units to show the problem.




The model shows that:

4 units  $\rightarrow$  315 mL

1 unit  $\rightarrow 315 \div 4 = \frac{\quad}{\quad}$  mL

6 units  $\rightarrow 6 \times \frac{\quad}{\quad} = \frac{\quad}{\quad}$  mL

$\frac{\quad}{\quad}$  milliliters of the mixture is carrot juice.

$\frac{1}{3}$  of  $\frac{\quad}{\quad}$  units  
 $= \frac{1}{3} \times \frac{\quad}{\quad}$  units  
 $= \frac{\quad}{\quad}$  units  
 $\frac{2}{5}$  of  $\frac{\quad}{\quad}$  units  
 $= \frac{2}{5} \times \frac{\quad}{\quad}$  units  
 $= \frac{\quad}{\quad}$  units

## 5th Grade Enrichment

A tiger is  $\frac{7}{8}$  meter tall. A koala bear is  $\frac{4}{7}$  as tall as the tiger. A penguin is  $\frac{7}{8}$  as tall as the koala bear. What is the height of the penguin?

Write your answer as a fraction.



## 5th Grade Enrichment

A rectangular tank is  $\frac{5}{9}$  full of water. If  $6\frac{3}{4}$  liters of water are poured into the tank, it will be  $\frac{2}{3}$  full. What is the capacity of the tank?

